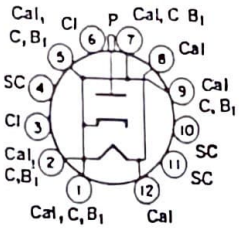
Ver tabla al final de esta sección. **2BN4**
Ver tipo 6BN4A. **2BN4A**



RECTIFICADOR DE MEDIA ONDA DE VACIO

2BU2
2BU2/
2AH2

Tipo duodecar utilizado como rectificador de alta tensión para alimentar al ánodo del tubo de imagen en receptores de TV. Sección *Dimensiones*, 9B; requiere zócalo de 12 contactos. Los terminales 4, 10 y 11 pueden utilizarse como puentes de conexión para componentes con potenciales iguales o cercanos al de calefactor. Para precauciones en alta tensión y rayos X, ver página 112. Tensión de calefactor (c.a./c.c.), 2,5 V; 0,33 ampere.

RECTIFICADOR PARA FLYBACK

Para funcionamiento en un sistema de 525 líneas, 30 cuadros

Regímenes máximos *(valores máximos de diseño)*:

Tensión de cresta de placa*	30000*	volts
Corriente de cresta de placa	80	mA
Corriente media de placa	1,5	mA
Tensión de calefactor:		
Valor máximo absoluto**	2,9	volts
Valor mínimo absoluto	2,1	volts

Características: *(valores instantáneos)*:

Caída de tensión de la válvula (aprox.) para corriente de placa de 7 mA	60	volts

Características de radiación X:

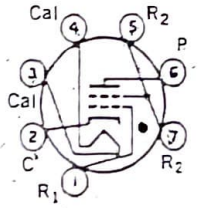
Radiación X, máxima:		
Valor estadístico controlado sobre la base de prueba de muestras	0,5	mR/h

* La duración del pulso de tensión no debe exceder el 15 % de un ciclo de exploración horizontal (10 microsegundos).

• La componente de c.c. no deberá exceder de 24000 volts.

** El funcionamiento de esta válvula durante un período prolongado de tiempo, con tensión de filamento superior al valor absoluto especificado de 2,9 V, podría resultar en la producción de radiación X superior al máximo límite especificado de 0,5 mR/h.

Ver tabla al final de esta sección. **2CN3A**
Ver tipo 6CW4. **2CW4**
Ver tipo 6CY5. **2CY5**



TIRATRON GASEOSO

2D21
TIPO INDUSTRIAL

Tiratrón tetródico gaseoso, tipo miniatura, para usar en relés. Sección *Dimensiones*, 5C; requiere zócalo miniatura de 7 contactos.

Tensión de calefactor (c.a./c.c.)	6,3 ± 10 %	volts
Corriente de calefactor	0.6	ampere
Cátodo:		
Tiempo de calentamiento previo a la conducción de la válvula	10	segundos
Tensión entre calefactor y cátodo:		
Valor de cresta	+ 100 — 25	volts

UNIVERSIDAD NACIONAL AUTONOMA DE MEXICO

FACULTAD DE CIENCIAS
LABORATORIO DE FISICA MODERNA
EXPERIMENTO PRELIMINAR 2o SEMESTRE

POTENCIAL DE IONIZACION

REFERENCIAS

1.- H.E. White, "Introduction to Atomic Spectra", Mc Graw-Hill, N.Y. (1934)
2.- P.W. Atkins, "Molecular Quantum Mechanics", V. II, Clarendon Press, Oxford (1970)
3.- G. Herzberg, "Atomic Spectra and Atomic Structure", Dover Publ., N.Y. (1945)
4.- H. Selig et. al., "The Chemistry of the Noble Gases", Sci. Amer. V. 210, Mayo, 66 (1964)

PROPOSITO

Mediante un método eléctrico, medir la energía (o potencial) de ionización de un gas, utilizando un tubo thyratrón.

INTRODUCCION

Las propiedades de los átomos están determinadas principalmente por su estructura electrónica. El número atómico Z representa una caracteristica fundamental del átomo, que determina en alto grado sus propiedades. La disposición de los átomos en la tabla periódica es precisamente una clasificación según el orden creciente de los números atómicos. La perioricidad se puede apreciar en forma sorprendente si se grafican las propiedades de los átomos como función de Z (véase la figura 1). Los potenciales de ionización son un buen ejemplo de esto, ya que aumentan monotonamente en cada periodo de la tabla alcanzando el valor máximo en un átomo de gás noble y disminuyendo en el período siguiente. La magnitud del potencial de ionización caracteriza la energía de enlace de los electrones orbitales del átomo. Se denomina primera energía de ionización a la minima energía necesaria para ssacar un electrón de un átomo neutro. Si se aplican energías mayores, es posible continuar sacando electrones del átomo, a estas se les llama segunda, tercera, ... energía de ionización.

INDICACIONES

Para este experimento se usa un tubo electrónico comercial (bulbo) lleno de gas a baja presión. Esquematicamente el tubo es como muestra la figura 3.

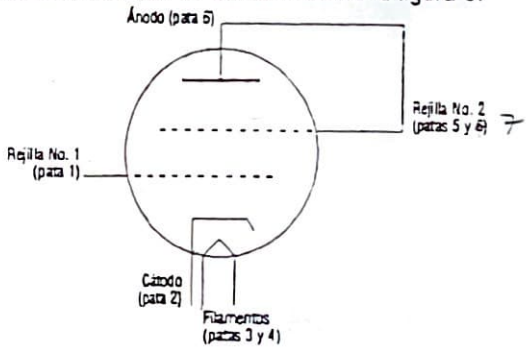


Fig. 3 Esquema del tubo 2021. La conexión entre la rejilla No. 2 y el ánodo es exterior (hágala usted)

Cuando el filamento calienta al cátodo, éste emite electrones térmicos los que pueden acelerarse aplicando una diferencia de potencial entre el cátodo y una de las rejillas. La otra rejilla puede usarse para controlar el número de electrones que llegan al ánodo. En el ánodo se colectan los electrones, que pueden detectarse entonces como una corriente eléctrica. La idea del experimento es variar la diferencia de potencial entre el ánodo y el cátodo hasta que el gas se ionice. Cuando ésto ocurre la corriente que registra el ánodo se incrementa subitamente, a la vez que el voltaje cae. La razón de esto puede verse como sigue: la corriente que circula depende del volyaje de acuerdo a la ecuación de Langmuir-Child

$$I = k V^{3/2}$$

con k una constante. pero el incremento de la corriente con el voltaje solo sigue esta ley hasta alcanzar el potencial de ionización de los átomos del gas; en este punto la corriente se incrementa bruscamente debido a la aparición de más cargas libres producto de la ionización, lo que produce a su vez un cambio de pendiente en la curva de $I^{2/3}$ vs. V. este comportamiento es el que nos permite determinar el primer potencial de ionización.

1) Diseñe un circuito simple que permita medir el potencial de ionización del gas, **tenga cuidado de no rebasar las especificaciones de los tubos.** Vea las hojas de características anexas.

2) Para reducir los efectos de la carga espacial (La nube de iones se forma cerca del ánodo) es conveniente poner la segunda rejilla al mismo potencial del ánodo.

El proceso de ionización de un átomo neutro puede considerarse como una transición entre dos estados del sistema; el estado inicial corresponde al estado base del átomo y el estado final correspondiente a un ión positivo por un lado y a un electrón en el continuo con energía cinética cero. La figura 2, representa esquematicamente esta transición. La diferencia de energía E_i es la energía que debe suministrarse al átomo neutro para efectuar la transición.

En este experimento se determinará el primer potencial de ionización de un átomo usando un método muy simple. Se emplea un tubo que contiene un gas a baja presión y se hacen incidir electrones de energía cinética conocida sobre el gas.

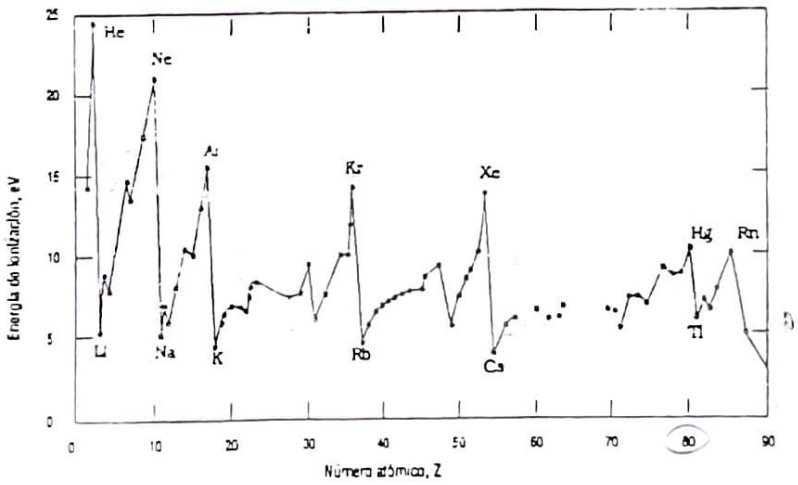


Fig. 1 Energía de ionización de los elementos en función del número atómico

Los electrones de energías bajas chocarán con los átomos del gas, pero los electrones que tengan energías por arriba del umbral de ionización (i.e. Ecin Ei) chocarán inelásticamente y provocarán que otro electrón sea expulsado del átomo, ionizándolo.

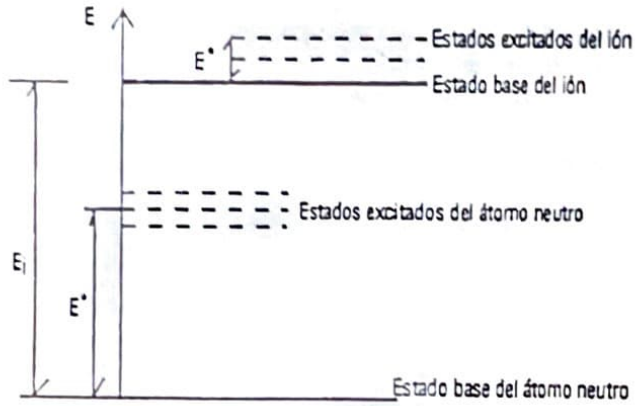


Fig. 2 Diagrama de niveles de energía para el átomo y el ión positivo al remover un electrón.

3) es recomendable que la temperatura del cátodo sea la más alta posible, para tener el número máximo de electrones emitidos por unidad de área. Nuevamente tenga cuidado de no rebasar las especificaciones del tubo.

PREGUNTAS

1.- ¿Cuál es el gas encerrado en el tubo?
2.- Con este método simple se podría determinar el segundo potencial de ionización? ¿Por que?
3.- Discuta brevemente al menos otro método para determinar el potencial de ionización.
4.- ¿Como se podría emplear este tubo para determinar energías de excitación de los átomos del gas (experimento de Frank-Hertz)? ¿Se aprecia esto en sus datos?
5.- ¿Como explica usted que mientras los potenciales de ionización son máximos para los gases nobles, estos resultados mínimos para los alcalinos?

René Ortega

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Space-Charge-Limited Currents in Solids

A. Rose
RCA Laboratories, Princeton, New Jersey
(Received September 21, 1954)

Currents, far in excess of ohmic currents, can be drawn through thin, relatively perfect insulating crystals. These currents are the direct analog of space-charge-limited currents in a vacuum diode. In actual crystals, the space-charge-limited currents are less than their theoretical value for an ideal crystal by the ratio of free to trapped carriers. Space-charge-limited currents become, therefore, a simple tool for measuring the imperfections in crystals even in the range of one part in 10^{16}.

The presence of traps not only reduces the magnitude of space-charge-limited currents, but also is likely to distort the shape of the current-voltage curve from an ideal square law to a much higher power dependence on voltage. The particular shape can be used to determine the energy distribution of traps.

The presence of traps tends to uniformize the charge distribution between electrodes, to introduce a temperature dependence of the current, and to give rise to certain transient effects from which capture cross sections of traps may be computed.

Space-charge-limited currents offer another mechanism for electrical breakdown in insulators.

I. INTRODUCTION

THE solid state analog of space-charge-limited currents in a vacuum diode are the space-charge-limited currents in an insulator. This was clearly pointed out at least fifteen years ago as a simple consequence of the band theory of solids.[1]

While there have been many references, as in the work of Hilsch, Gudden, and Pohl,[2] to the transient effects of space charge in solids, there have not been until recently direct measurements of steady-state space-charge-limited currents.[3–6] The lack of such measurements is remarkable since simple theory allows amperes per square centimeter of space-charge-limited current to be passed through thin sheets of insulators. Two requirements, however, need to be fulfilled in order to observe space-charge-limited currents of significant magnitude: At least one of the two electrodes must take ohmic contact[5,7] to the insulator and the insulator must be relatively free from trapping defects. The concept of an ohmic contact to an insulator is perhaps not a common one and needs to be defined. An ohmic contact is used here to mean an electrode that supplies an excess or a reservoir of carriers ready to enter the insulator as needed. The virtual cathode formed in front of a thermionic emitter in a vacuum diode is a familiar example of an ohmic contact to the insulating vacuum space between cathode and anode.[8] The current through the vacuum diode or between electrodes in an insulating solid does not depend on the amount of excess carriers as long as there is an excess.

Figure 1 shows one example of an ohmic contact to an insulator obtained by the use of a metal whose work function is less than that of the insulator. The presence of the virtual cathode is evident in Fig. 1(b).

The requirement of relative freedom from traps will be made quantitative later. For the present, it is sufficient to point out that traps lower the drift mobility of carriers and thereby the magnitude of the space-charge-limited currents.[4] Trap densities of $10^{14}/cm^3$ (not unreasonable for the usual polycrystalline insulator) would be sufficient to reduce the space-charge-limited currents to almost unmeasurable values.

The measurements of space-charge-limited currents reported by Smith[9] are on relatively perfect insulating

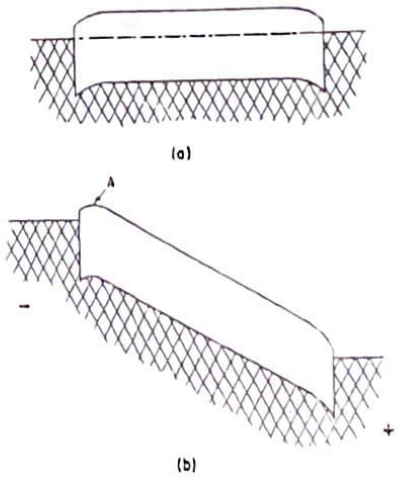

FIG. 1. (a) Ohmic contacts to an insulator at zero applied field. (b) Finite field applied to Fig. 1(a) showing virtual cathode at A.

[1] N. F. Mott and R. W. Gurney, *Electronic Processes in Ionic Crystals* (Oxford University Press, New York, 1940), p. 172.
[2] B. Gudden, *Lichtelektrische Erscheinungen* (Verlag Julius Springer, Berlin, 1928).
[3] P. K. Weimer and A. D. Cope, RCA Review 12, 314 (1951).
[4] A. Rose, RCA Review 12, 362 (1951).
[5] R. W. Smith and A. Rose, Phys. Rev. 92, 857 (1953); A. Rose and R. W. Smith, Phys. Rev. 92, 857 (1953).
[6] W. Shockley and R. C. Prim, Phys. Rev. 90, 753 (1953); G. C. Dacey, Phys. Rev. 90, 759 (1953).
[7] R. W. Smith, this issue [Phys. Rev. 97, 1525 (1955)].
[8] L. S. Nergaard [RCA Rev. 13, 464 (1952)] proposes a model of an oxide cathode in which the flow of current within the cathode coating itself, as well as in the vacuum just outside the cathode, may be space-charge-limited.
[9] R. W. Smith and A. Rose, preceding paper [Phys. Rev. 97, 1531 (1955)].

crystals of CdS having ohmic contacts. Even so, there are a sufficient number of traps that the simple model for space-charge-limited currents needs to be modified as in the following analysis to take their effect into account. The traps not only reduce the magnitude of the space-charge-limited current, but also distort the shape of the current-voltage curve and add certain interesting and informative transient effects.

The analysis of space-charge-limited currents is carried out in terms of the following approximate but simple formalism. Let the space between two electrodes have a capacitance C. This is an approximating concept. In the case of plane parallel electrodes the capacitance is that between the two electrodes. The charge that can be accommodated in the interior space is

$$Q=CV, \tag{1}$$

where V is the applied voltage.

The space-charge-limited current is immediately given by

$$I=Q/T, \tag{2}$$

where T is the transit time of the charge Q between electrodes.

The well-known expressions for space-charge-limited currents in vacuum and in a trap-free insulator are readily derivable from Eq. (2). They are given here to clarify the formalism.

II. VACUUM DIODE

The space charge forced into the vacuum diode per cm² of plate area and for a plate separation of d cm is

$$Q=CV=(V/4\pi d)\times 10^{-12} \text{ coulomb}. \tag{3}$$

The transit time of the charge Q between plates is approximately

$$T=d/(6\times 10^{7}\times V^{\frac{1}{2}}) \text{ sec}. \tag{4}$$

The space-charge-limited current is, from (2), (3), and (4)

$$I=5\times 10^{-6}(V^{\frac{1}{2}}/d^{2}) \text{ amperes/cm}^{2}. \tag{5}$$

The accurate value of the coefficient is 2.3×10^{-6}.

III. TRAP-FREE INSULATOR

The space charge forced into an insulator per cm² of plate area is, from Eq. (1)

$$Q=(Vk/4\pi d)\times 10^{-12} \text{ coulomb}; \tag{6}$$

k is the dielectric constant of the insulator and d the electrode spacing. The transit time of the charge Q between electrodes is

$$T=d/E\mu=d^{2}/V\mu. \tag{7}$$

E is the electric field in the insulator and μ the drift mobility. From Eqs. (2), (6), and (7) the space-charge-limited current is

$$I=10^{-13}(V^{2}\mu k/d^{3}) \text{ amperes/cm}^{2}. \tag{8}$$

The accurate value of the coefficient is also 10^{-13}.

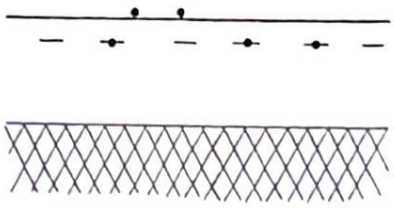

FIG. 2. Insulator having shallow traps in thermal equilibrium with electrons in the conduction band.

IV. INSULATOR WITH SHALLOW TRAPS

Let the insulator have only shallow traps (Fig. 2), that is, traps lying close enought to the conduction band to be in thermal equilibrium with electrons in the conduction band. The same expression for the space-charge-limited current will be obtained as in the case of the trap-free insulator. One need only insert for the drift mobility the product of the drift mobility for free carriers and the fraction of the total space-charge that is free. While the same total charge is forced into the insulator as in the case of the trap-free insulator, only a fraction of this charge is free. The drift mobility must be reduced by the same fraction. The value of this fraction is determined by the number and depth of traps and is *not dependent on the applied voltage*.[10] Accordingly, the space-charge-limited current has the same square-law dependence on voltage as in the simple trap-free model of Eq. (8).

Let the fraction of free charge be θ. The space-charge-limited current is then given by

$$I=10^{-13}[V^{2}(\mu_0\theta)k/d^{3}] \text{ amperes/cm}^{2}, \tag{9}$$

where μ_0 is the drift mobility of *free* carriers.

If there is a single level of shallow traps whose density is N_t/cm³ and whose distance from the conduction band is E volts, the fraction θ is given at room temperature by the approximate relation

$$\theta=(N_c/N_t)e^{-E/kT}, \tag{10}$$

where $N_c=10^{19}$ at room temperature. For $N_t=10^{17}$ and $E=0.5$ volt, $\theta=10^{-7}$ and the space-charge-limited currents are sharply reduced.

V. INSULATOR WITH TRAPS DISTRIBUTED IN ENERGY

Consider, as shown in Fig. 3, an insulator in which the traps are distributed uniformly in energy below the conduction band. The prominent characteristics of this model are a consequence of the distribution of

[10] The electron temperature is assumed here to be the same as the crystal temperature.

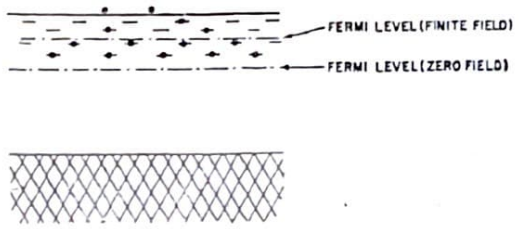

FIG. 3. Insulator having a distribution of traps in energy and showing the shift in Fermi level due to charge injected by an applied field.

traps in energy and not of the strict uniformity of distribution. For a given applied voltage, the charge, Q, forced into the insulator, is distributed in three major parts: free charge in the conduction band, trapped charge above the newly determined Fermi level, and trapped charge condensed in the states between the original Fermi level and the newly determined Fermi level. Since the condensed charge is likely to be very nearly the total charge, the new location of the Fermi level is given very closely by considering all of the charge Q to be condensed. With this approximation, the shift in Fermi level will be proportional to the space charge Q which is, in turn, proportional to the applied voltage V.

We can write for the free carrier density

$$n_c = N_c e^{-E_f/kT} e^{\Delta E/kT}. \tag{11}$$

Here N_c is the number of states in the bottom kT slice of the conduction band, E_f is the original distance of the Fermi level from the conduction band and ΔE is the shift in position of the Fermi level owing to the condensed charge Q forced into the insulator by the applied voltage V. Also, from previous remarks:

$$\Delta E = Q/en_t d = VC/en_t d, \tag{12}$$

where n_t is the number of traps per cm³ per unit range in energy and e the electron charge. From Eqs. (11) and (12) the free carrier density is given by

$$n_c = N_c e^{-E_f/kT} e^{VC/n_t dekT} \tag{13a}$$

$$= n_{c0} e^{\alpha V}, \tag{13b}$$

where n_{c0} is the initial, thermal equilibrium concentration of free carriers and α is used for $C/n_t dekT$.

The density of trapped carriers is very nearly equal to the total density of injected electrons, or

$$\text{density of trapped carriers} = Q/de = VC/de. \tag{14}$$

The fractional value of free charge is, from Eqs. (13) and (14),

$$\theta = (en_{c0}d/VC)e^{\alpha V}. \tag{15}$$

θ is no longer a constant as in the previous case of shallow traps, but depends exponentially on the applied voltage. From Eqs. (9) and (15) the space-charge-limited current becomes:

$$I = 10^{-12}(V\mu_0 k/d^2)(en_{c0}/C)e^{\alpha V}. \tag{16}$$

What is significant in Eq. (16) is that, owing to the distribution of traps in energy, the space-charge-limited current now increases exponentially with voltage compared with the square law dependence on voltage obtained in the trap-free and in the shallow-trap models. The exponential dependence is a consequence of the assumption of a *uniform* distribution of traps. If the uniform distribution of traps is replaced by one that decreases with distance from the conduction band, the exponential is replaced by a high power function of the voltage.

In particular, let the steepness of the trap distribution be approximated by a characteristic temperature T_c such that

$$n_t \propto e^{-E/kT_c}, \tag{17}$$

where E is measured from the bottom of the conduction band. Small values of T_c lead to trap distributions varying rapidly with energy, while large values of T_c approximate a slowly varying trap distribution. The voltage dependence (see Appendix I) of space-charge-limited current is (for $T_c > T$)

$$I \propto V^{(T_c/T)+1}. \tag{18}$$

For $T_c < T$, this reduces to the case of shallow traps where the exponent of V is 2.

VI. TRAP DISTRIBUTION FROM I VS V CURVE

One can expect to work backwards from an experimentally determined current-voltage curve to obtain the energy distribution of traps. Equation (18), for example, gives the trap distribution for experimental curves for which the current increases as a power of the voltage. For a current-voltage curve of arbitrary form, and for currents increasing faster than V^2, the following analysis may be made. From Eq. (9) one may write

$$I = \text{constant} V e^{\Delta E/kT}, \tag{19}$$

and

$$\frac{dI}{dV} = \frac{I}{V}\left(1 + \frac{V}{kT}\frac{d(\Delta E)}{dV}\right). \tag{20}$$

The solution of Eq. (20) for $d\Delta E/dV$ is

$$\frac{d\Delta E}{dV} = \left(\frac{V}{I}\frac{dI}{dV} - 1\right)\frac{kT}{V}. \tag{21}$$

Since the charge condensed in traps is

$$Q = VC,$$

Eq. (21) may be rewritten as

$$C\frac{d\Delta E}{dQ} = \left(\frac{V}{I}\frac{dI}{dV} - 1\right)\frac{kT}{V}, \tag{22}$$

or

$$\frac{dQ}{d\Delta E}=\frac{CV}{kT}\left(\frac{V}{I}\frac{dI}{dV}-1\right)^{-1}. \tag{23}$$

In Eq. (23), $dQ/ed\Delta E$ is the number of traps per unit energy range in the volume of specimen under test. A simple operational interpretation of Eq. (23) is the following. If one increases the applied voltage by an amount ΔV sufficient to double the current, the number of electron charges forced into the insulator is $\Delta VC/e$. This number is also the number of traps in a range kT near the Fermi level.

VII. COMPARISON OF SPACE-CHARGE-LIMITED CURRENT WITH PHOTOCONDUCTIVE CURRENT

The last two sections have shown how the form and magnitude of the trap distribution may be computed from the space-charge-limited current-voltage curve.

TABLE I. The energy distribution and density of traps derived from data on photoconductivity and from data on space-charge-limited currents.[a]

	Space-charge-limited currents	Photoconductivity
Form of current curve	$I\propto V^{(T_c+T)/T}$	$I\propto F\tau_\sigma/(T+T_c)$
Trap density in range of kT near the Fermi level	$\Delta Q/e$	$(\tau_\sigma/\tau)n_c$

[a] Notes: T_c defines the trap distribution by Eq. (17). F is the number of optical excitations per second. ΔQ is the charge forced into the insulator when the voltage is increased by an amount sufficient to double the current. τ_σ is the observed response time of the photoconductor to interrupted light. τ is the lifetime of a free carrier in the conduction band. n_c is the density of free carriers at which τ_σ is measured. The Fermi level is defined by the relation: $n_c=N_ce^{-E_f/kT}=10^{16}e^{-E_f/kT}$.

In reference 4, it was argued that the same information on trap distribution could be obtained from data on the form of the photocurrent-light curve and from data on the ratio of lifetime to observed time constant.

The results of the two analyses are summarized in Table I.

The characteristic temperature, computed from the space-charge-limited currents, should be more reliable than the characteristic temperature computed from the photoconductive currents. In the analysis of the latter an implicit assumption was made that all of the traps had the same capture cross section for electrons. The validity of this assumption is under study and must, in any event, be tested for each new crystal. There are some observations that require the presence of more than one type of trap and such mixtures can alter the form of the current-light curve. The form of the space-charge-limited current-voltage curve on the other hand should not be dependent on the capture cross section of the traps. Reasonable agreement between the two independent methods of measuring trap distributions is reported by Smith and Rose[9] and by Bube.[11]

[11] R. H. Bube and S. M. Thomsen, J. Chem. Phys. 23, 15 (1955).

VIII. FIELD AND CHARGE DISTRIBUTION BETWEEN ELECTRODES

For the simple case of trap-free insulator and plane parallel electrodes, the following relations for current, field, and charge distribution are known:

$$I\propto V^2 \tag{24}$$

$$E\propto x^1 \tag{25}$$

$$\rho\propto x^{-1}, \tag{26}$$

where E is the electric field, x the distance from the cathode (for electron injection), and ρ the space charge density.

In Appendix II it is shown that, in general, when traps are present and when

$$I\propto V^{n+1},\quad n\geq 1, \tag{27}$$

the field and charge distributions take on the forms

$$E\propto x^{n/(n+1)}, \tag{28}$$

$$\rho\propto x^{-1/(n+1)}. \tag{29}$$

For large values of n, the space charge density approaches a uniform distribution over most of the distance between cathode and anode. The free charge density, however, must always vary as the reciprocal of the field in order to keep the divergence of the current zero. Since the free charge is usually a negligible part of the total charge, it may undergo large variations without having significant effect on the distribution of the total charge.

The relative uniformity of charge density between cathode and anode leads one to expect only small or negligible currents when these electrodes are shorted together. The space charge flowing out of the insulator tends to flow out equally at both ends. Smith[9] has observed the short circuit current to be negligibly small. This is to be contrasted with the relatively large short-circuit reverse currents obtained from dielectric absorption effects as in some glasses.

IX. TRANSIENT EFFECTS

The following observation on space-charge-limited currents in CdS crystals is reported by Smith.[9] A sudden increase in voltage causes the current to transiently increase to very high values. In a matter of seconds or minutes the current subsides to a much smaller stationary value. The interpretation is that the sudden increase in voltage forced a corresponding increase of charge in the *conduction band*. In the course of seconds, most of this free charge settles into traps and one observes the rapid decay of current. The time required for the transient current to subside is a direct measure of the capture cross section of traps for free electrons.

If the space-charge-limited current has attained a stationary value at a given voltage it is found[9] that lowering the voltage from this value may cause the

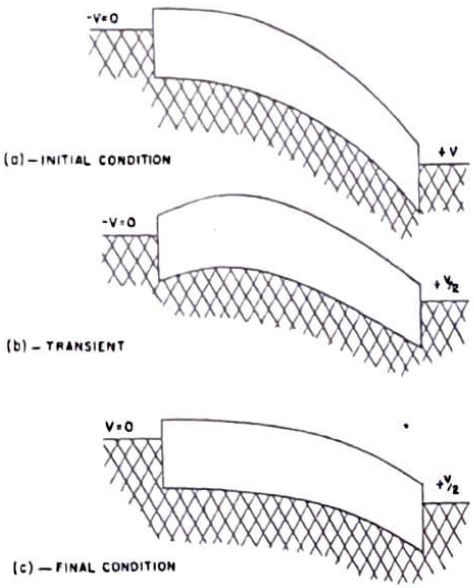

FIG. 4. Series of potential patterns showing the transient effects when the applied voltage is reduced.

current to "undershoot" its new stationary value. The interpretation here is shown in Fig. 4. Figure 4(a) shows the stationary potential distribution at an applied voltage V_1. When the voltage is lowered to $\frac{1}{2}V_1$, it requires some time for the trapped space charge forced into the crystal at V_1 to be thermally released. Before this trapped charge is thermally released, a space-charge barrier is presented to the cathode, as shown in Fig. 4(b). *This is not a virtual cathode type of barrier.* It actually *suppresses* the entrance of electrons from the cathode into the insulator. As time goes on, the trapped charge is thermally released and the potential distribution arrives at the new stationary value shown in Fig. 4(c). If the thermal release of carriers is sufficiently slow (traps of small capture cross section) the current can "undershoot" its final value. If the thermal release is fast there will be no "undershoot" but actually an "overshoot."

X. TEMPERATURE DEPENDENCE

The injection of space charge into an insulator converts it into a semiconductor of increasing conductivity with increasing voltage. At any given voltage the current should vary with temperature as would any semiconductor having the same conductivity. (This does not mean that the temperature variation of conductivity is determined only by the conductivity. As in any semiconductor the trap distribution governs the temperature dependence.) An increase in temperature does not alter the total amount of space charge, but does increase the fraction of this space charge in the conduction band.

Equation (18) indicates that lowering the temperature should make the current-voltage curve steeper. For very steep curves, the effect of lowering the temperature should be one of shifting the current-voltage curve along the voltage axis toward higher voltages. To match the same current the Fermi level must be closer to the conduction band at lower temperatures and this requires higher voltages according to Eq. (12).

XI. TRANSITION FROM OHMIC TO SPACE-CHARGE-LIMITED CURRENTS

Space-charge-limited currents increase as the square or as some higher power of the voltage. Ohmic currents increase linearly with the voltage. One would expect, therefore, that for any finite conductivity, there would be a range of voltages near zero for which the ohmic currents would predominate. For voltages higher than some critical voltage, space-charge-limited currents would predominate. The critical voltage at which this transition from ohmic to space-charge-limited behavior takes place should increase as the normal *volume-generated* conductivity increases. Results of this character are clearly reported in reference 9 where the critical voltage is varied by shining light on a CdS crystal.

What has just been described should certainly take place if the ohmic and space-charge-limited currents were in parallel, physically separate paths. When the two types of current occupy the same physical volume, the transition from one current to the other is likely to be somewhat more involved because the potential distributions are different for the two types of current. There will then be a competition between the two processes to establish their appropriate potential distribution. It would appear, however, from qualitative arguments that the mechanism that introduced the larger density of free carriers would control the potential distribution. Accordingly, higher volume generated carrier densities mean that a higher voltage is required before the injected space-charge-carrier densities predominate and determine the character of the current-voltage curve.

It is interesting that even in the range of voltage where the ohmic currents predominate in the steady state, the space-charge-limited currents may determine the transient behavior. This follows from the fact that when the voltage is increased there is a transient high density of space-charge-carriers in the conduction band —a density that may exceed that of the volume generated carriers. As these space-charge-carriers become trapped, their density falls below that of the volume-generated carriers and the latter lead to steady-state ohmic currents.

XII. TOOL FOR MEASURING CRYSTAL DEFECTS

As already outlined in an earlier section, the number and energy distribution of traps can be deduced from

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the current-voltage curve for space-charge-limited currents. What needs to be emphasized here is that these currents constitute an unusual tool for measuring defect structure—a tool that becomes particularly effective in the range of low concentration of defects.

The effect of traps is generally to reduce the observed space-charge-limited currents below their theoretical value for a trap-free crystal. The measure of this reduction is the ratio of free to trapped carriers. Thus, the observed currents should approach those for a perfect crystal when the number of free carriers matches or exceeds the number of traps. The more perfect the crystal, the lower the field at which this occurs.

The density of electron charges forced into a crystal d centimeters thick, having plane parallel electrodes, may be written approximately as $10^7(V/d^2)$ electron charges/cm³ for an assumed dielectric constant of 10. This means that for a millimeter thick crystal having 10^{16} traps/cm³, the space-charge-limited currents should approach their theoretical values at ten volts. At this voltage the current will be about 10 microamperes/cm² for a mobility of 100 cm²/volt-sec. The measurement of trap densities of only one part in 10^{15} becomes then a simple current-voltage measurement at low voltages and easily measurable currents.

XIII. CONCERNING BREAKDOWN IN INSULATORS

When the voltage across an insulator is increased steadily the power dissipation in the insulator is finally increased to the point where the insulator "burns out" or is said to "break down." If enough carriers are normally present in the insulator, the breakdown is a relatively slow and gradual process in which the increased voltage at first leads to an increased temperature. More carriers are generated at the higher temperature and the approach to breakdown becomes more rapid. This process is known as "thermal breakdown" and may be followed reversibly to values close to actual breakdown. A second process[12] that has received the major share of theoretical attention is a fast, electronic process known as "intrinsic breakdown." Here a critical electric field may be observed at which the carrier density is precipitously increased by a collision ionization and resulting avalanching process or by field emission from the filled band. Even though the actual breakdown field has a sharply defined value, there is reason to expect in this model also that the prebreakdown currents will increase faster than linearly with voltage.

The present discussion adds a third mechanism for increasing the carrier density in insulators and must be considered in analyzing breakdown data. This mechanism of space-charge-limited currents becomes more significant as the crystallinity of the insulator improves. It may be distinguished from intrinsic breakdown by the fact that the breakdown field should increase approximately linearly with electrode spacing.

[12] H. Fröhlich and J. H. Simpson, *Advances in Electronics* (Academic Press, New York, 1950), Vol. 2, p. 185.

A rough estimate of the contribution of space-charge-limited currents may be made as follows. Let the intrinsic breakdown field strength be known. From this value of field and the known geometry of the specimen a value for the space-charge density in the insulator may be computed. This value of space charge density, converted to carrier density, must be comparable with the trap density in order that space-charge-limited currents be significant. For example, at a field of 10^5 volts/cm in an insulator 10^{-2} cm thick, the number of electron charges per cm³ forced into the insulator would be 10^{15}. Trap densities less than this value would allow space-charge-limited currents to be significant; trap densities greater than this value would tend to suppress the space-charge-limited currents.

APPENDIX

I. CURRENT-VOLTAGE CURVE FOR EXPONENTIAL TRAP DISTRIBUTIONS

The trap density per unit energy range is defined by

$$n_t = Ae^{-E/kT_c}, \tag{30}$$

where E is the energy measured from the bottom of the conduction band and T_c is a characteristic temperature greater than the temperature at which the currents are measured. The condensed charge forced into the insulator is

$$Q = VC. \tag{31}$$

This condensed charge raises the Fermi level by an amount ΔE defined by the relation

$$\int_{E_f-\Delta E}^{E_f} n_t dE = \frac{Q}{e} = \frac{VC}{e}, \tag{32}$$

or

$$\int_{E_f-\Delta E}^{E_f} Ae^{-E/kT_c} dE = \frac{VC}{e}. \tag{33}$$

The solution of Eq. (33), neglecting the upper limit of integration, is of the form

$$\Delta E = kT_c(K + \ln V), \tag{34}$$

where K contains the temperature but not the voltage. The ratio of free to trapped charge is [see Eq. (11)]

$$\theta = en_{c0}e^{\Delta E/kT}/VC. \tag{35}$$

If Eq. (34) is used for ΔE,

$$\theta = \text{constant} \exp[(T_c/T)\ln V]/V \tag{36}$$

$$= \text{constant} V^{(T_c/T)-1}. \tag{37}$$

This value for θ is now inserted in Eq. (9) to give

$$I \propto V^{(T_c/T)+1}. \tag{38}$$

II. FIELD AND CHARGE DISTRIBUTION BETWEEN CATHODE AND ANODE

A solution is sought for the usual pair of equations for one-dimensional space-charge-limited flow:

$$\frac{dE}{dx}=\frac{4\pi\rho}{k}=\frac{4\pi}{k}(\rho_f+\rho_t), \tag{39}$$

$$I=\rho_f\mu E, \tag{40}$$

subject to the boundary condition $E=0$ at $x=0$. ρ is the space charge density and is composed of a part, ρ_f, in the conduction band and a part, ρ_t, in traps. Let $\rho_f \ll \rho_t$ so that it may be neglected in Eq. (39). Also, from Eq. (37) let

$$\rho_f=A\rho_t^n, \tag{41}$$

where

$$n=T_c/T. \tag{42}$$

Equation (39) may be rewritten, using Eqs. (40) and (41), in the form

$$\frac{dE}{dx}=\frac{4\pi}{k}\left(\frac{\rho_f}{A}\right)^{1/n}=\frac{4\pi}{k}\left(\frac{I}{\mu EA}\right)^{1/n}=BE^{-1/n}, \tag{43}$$

where $B=(4\pi/k)(I/\mu A)^{1/n}$.

The solution of Eq. (43) satisfying the boundary condition is

$$E=[(n+1)/n]Bx^{n/(n+1)}. \tag{44}$$

As $n\to\infty$, $E\to Bx$. If $n=1$, the usual form for a trap-free (as well as a shallow trap) model is obtained, namely $E\propto x^{\frac{1}{2}}$.

The distribution of trapped space charge using Eq. (39) is

$$\rho_t=\frac{k}{4\pi}\frac{dE}{dx}$$

$$=\frac{k}{4\pi}\frac{n+1}{n}Bx^{-1/(1+n)}. \tag{45}$$

Again this reduces to the familiar $x^{-\frac{1}{2}}$ form when $n=1$, *but approaches a constant for large n.*

The uniform distribution of space charge, at large n, means that when the electrodes are shorted only a vanishingly small net current will flow as the space charge leaves the insulator. The space charge will flow out almost symmetrically at both ends of the insulator.

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Equation of State of Metals from Shock Wave Measurements*

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(Received October 21, 1954)

Shock wave pressure magnitudes from about 150 to 500 kilobars have been attained for metals by using high explosives. A photographic technique for the nearly simultaneous determination of shock and free surface velocities is presented, and measurements for aluminum, copper, and zinc are given.

Expressions are derived which permit the calculation of pressure-compression points from measured velocity pairs. Consequent Hugoniot curves are presented, probable errors for which are 1 to 2 percent in compression for a given pressure. Finally, the known Hugoniot curves are employed in a calculation which determines temperatures and isotherms.

I. INTRODUCTION

WHEN a detonation wave interacts with an explosive-metal interface, a compression wave is transmitted into the metal. In the ordinary case this disturbance is a shock wave separating a compressed state from the undisturbed metal. The pressures attained behind such shock waves are typically in the range 150 to 500 kilobars (1 kilobar $=10^9$ dynes/cm^2 $=986.9$ atmospheres). The associated problem of determining pressure-compression data from shock wave measurements is the subject of the present investigation. Such data serve to supplement and extend the wealth of static pressure-compression data which exist for pressures up to 100 kilobars.[1]

Two basic assumptions are employed throughout the present considerations. First, since shock pressures are several hundred times yield points of the materials involved, an ordinary "fluid" type equation of state is assumed, i.e., a functional relationship (unspecified) between P, V, and T is assumed to be an adequate representation of the metal. This assumption precludes the explicit treatment of effects arising from the material rigidity which, however, are felt to play a

* Work done under the auspices of the U. S. Atomic Energy Commission. Papers on this subject were presented by the authors at the July, 1953 meeting of the Fluid Dynamics Section of the American Physical Society at State College, Pennsylvania and at the 1954 annual meeting of the American Physical Society.

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[1] See P. W. Bridgman, Revs. Modern Phys. 18, 1–93 (1946) for a general review.

Space-Charge-Limited Currents in Single Crystals of Cadmium Sulfide*

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(Received September 21, 1954)

Currents as high as 20 amperes per cm² can be drawn through thin insulating crystals of CdS in the dark. A series of experiments demonstrate with a high degree of certainty that these are space-charge-limited currents—the solid state analog of space-charge-limited currents in a vacuum. This conclusion is contrary to a recently published interpretation of similar observations on CdS crystals by Böer and Kümmel.

The use of pulsed voltages made possible the observation of currents close to those of a trap-free solid. The steady-state currents are many orders of magnitude lower than these but still many orders of magnitude higher than would be expected from the low-field resistivity of the insulator. The presence of traps determines the form and magnitude of the steady-state current-voltage curves. Conversely, these curves become a sensitive tool for the measurement of trap densities. Trap densities computed independently from space-charge-limited currents and from photoconductive currents show reasonable agreement.

INTRODUCTION

THE early band theory models of an insulator carried with them implicitly the suggestion that if free carriers could be injected into either the conduction band or the valence band, these carriers could move freely through the solid. The magnitude of current that could be passed through a "perfect" insulator would be limited only by the space charge of the carriers themselves, just as the space-charge-limited currents in a vacuum diode. Mott and Gurney[1] derived the relation:

$$I = 10^{-12} V^2 \mu k / d^3 \text{ amperes/cm}^2 \qquad (1)$$

for the space-charge-limited current I through a slab of insulator d centimeters thick when V volts were applied. μ and k are the drift mobility and dielectric constant respectively. It is interesting that this expression leads to the expectation of some tens of amperes per square centimeter through an insulating sheet 10^{-3} cm thick when ten volts are applied across opposite faces.

The literature not only does not bear out these large currents but is almost devoid of any evidence for steady-state space-charge-limited currents. Gudden,[2] and many others since, have described the *transient* effects of space charge in insulators, chiefly in suppressing photo- or bombardment-induced currents. Weimer and Cope[3] cite evidence for small photogenerated space-charge-limited currents thin films of amorphous selenium. The currents were of the order of 10^{-7} ampere/cm², but they were nevertheless *steady* currents. The present work describes the evidence for large, steady space-charge-limited currents drawn through thin insulating crystals of CdS by means of ohmic contacts.[4]

EARLY OBSERVATIONS

The first observations that led to identifying the space-charge-limited currents are shown in Fig. 1 which represents the V-I characteristics as seen on an oscilloscope. Sixty-cycle/sec ac voltages up to about 100 volts were applied across a thin ($\sim 5\times10^{-3}$ cm) insulating CdS crystal having indium electrodes. With two different values of light on the crystal, the two linear characteristics F_1 and F_2 were obtained. With the crystal in the dark and a small ac voltage applied, the curve a_1 was obtained. (If the amplitude of ac voltage is held fixed, the turned up ends of the a_1 curve tend to settle towards the voltage axis.) If the amplitude of ac voltage is increased toward V_1,

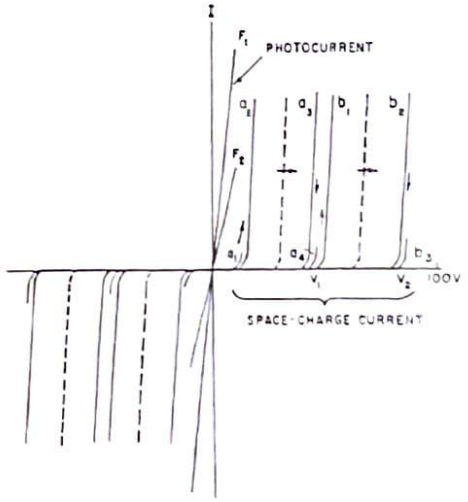

FIG. 1 Space-charge-limited current in an insulator. Sketch illustrating V-I characteristics, as seen on an oscilloscope, of insulating CdS crystal with *In* electrodes opposite one another on thin section of crystal. 60-cps voltage applied across crystal.

* Presented at the June, 1953, meeting of the American Physical Society [R. W. Smith and A. Rose, Phys. Rev. 92, 857(A) (1953); A. Rose and R. W. Smith, Phys. Rev. 92, 857(A) (1953)].

[1] N. F. Mott and R. W. Gurney, *Electronic Processes in Ionic Crystals* (Oxford University Press, London, 1940), pp. 168–173.

[2] B. Gudden, *Lichtelectrische Erscheinugen* (Verlag Julius Springer, Berlin, 1928).

[3] P. K. Weimer and A. D. Cope, RCA Rev. 12, 314 (1951).

[4] R. W. Smith, preceding paper [Phys. Rev. 97, 1525 (1955)].

the curve a_1 "shoots up" to the form a_2 and slides along the voltage axis with increasing amplitude of ac voltage. If the voltage amplitude is held fixed at V_1, the curve a_2 settles down to the form a_4.

At this point the amplitude of ac voltage may be reduced to zero and again increased toward V_1. Both during the decrease of voltage and during the increase of voltage the current remains vanishingly small until the voltage V_1 is reached at which point curve a_4 is retraced. Further increase of voltage from V_1 towards V_2 causes a_4 to "shoot up" again to b_1, slide along the voltage axis to b_2 and settle down to b_3 when the voltage amplitude is held fixed at V_2.

The interesting features of these V-I characteristics are the time dependence and the highly nonlinear but symmetric form of the curves.

The observation of a rapidly rising current-voltage curve is not in itself surprising. This is, indeed, a common observation in semiconductor measurements. And any one of several phenomena are commonly used to explain such curves. The phenomena include field emission from electrodes, from traps or from the valence band; collision ionization of trapped or valence electrons; poor contact; barriers; and heating effects. What *was* surprising was that none of these phenomena appeared to fit the observations.

The use of ohmic contacts[4,5] ruled out poor contacts and field emission from the electrodes. The linear current-voltage curve under illumination ruled out collision ionization. The low fields ($\sim 10^3$ volts/cm) ruled out field emission from traps or the valence band. The low currents ruled out heating effects. And finally, the assumption of internal barriers appeared improbable in the light of the otherwise regular performance of the crystal as a simple uniform photoconducting insulator.

In addition, the following effects were noteworthy: The high burst of current when the voltage was raised followed by a steady decline in current on standing; the increased resistance at a given voltage resulting from the previous application of higher voltage; the symmetry of the curves on the voltage axis.

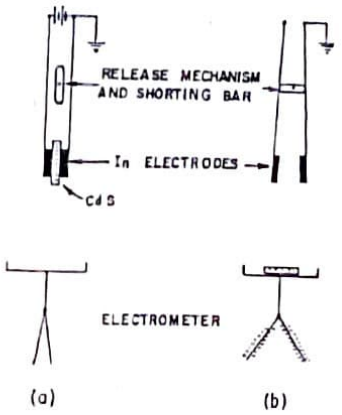

FIG. 2. Schematic drawing of experiment to measure charge injection in an insulator.

[4] C. I. Shulman (to be published).

QUALITATIVE INTERPRETATION

A quick appraisal of the expected properties of space-charge-limited currents in a solid, as modified by the presence of traps, indicated that most of the observations could be qualitatively accounted for by the assumption of such currents. These currents were all the more reasonable because the metal contacts were ohmic—that is, the contacts provided a reservoir, or excess of carriers, ready to enter the crystal as needed. A qualitative description of the properties of space-charge-limited currents in a solid follows.

When a voltage is first applied across the crystal, space charge, in the form of free electrons from the cathode, is forced into the crystal via its conduction band. This free electron charge gives rise to a large burst of current. If the space charge remained in the conduction band, the peak value of the transient current would continue as a steady current. In actual crystals, however, one must take into account the effects of trap densities of the order of 10^{16} cm^{-3}. The free charge forced into the conduction band settles into the traps, more or less rapidly, the rate being determined by the capture cross section of the traps. This accounts for the transient increase in current and subsequent decrease on standing.

For a given voltage across the crystal a fixed amount of charge is forced into the crystal; most of this charge becomes trapped and only a small fraction remains free. Because the trap density is likely to be large compared with the density of space-charge electrons, one can take as a first approximation that all of the charge is condensed into traps. The condensation of electrons in traps raises the Fermi level towards the conduction band. To preserve a proper statistical equilibrium, the density of electrons in the conduction band must now be increased in accordance with this shift in Fermi level. The second approximation is then to allocate some of the condensed or trapped charge to the conduction band. These two steps usually give a fairly accurate approximation as suggested by the following numerical example. Let the applied voltage force 10^{13} electron charges per cm^2 into the crystal. Let this result in raising the Fermi level 0.1 volt towards the conduction band. If the crystal were an insulator having 10^6 free electrons/cm^3 prior to the application of a voltage, it will now have about 10^8 free electrons per cm^3 to be consistent with the new position of the Fermi level. These 10^8 free electrons taken out of the 10^{13} condensed electrons will obviously have a negligible effect on relocating the Fermi level.

If the traps are more or less uniformly distributed in energy in the forbidden zone, equal increments in voltage will make equal energy shifts in the position of

the Fermi level. Since, however, the free electron density depends exponentially on the position of the Fermi level, the free electron density will increase exponentially with, or at least as some high power of, the applied voltage. This accounts for the observed high power dependence of current on voltage. By way of contrast, for a trap-free solid the current should increase only as the square of the applied voltage.

The space-charge-limited currents were also consistent with the observed low fields, there being no theoretical threshold field for the observation of space-charge-limited currents. Finally, it was reasonable to expect that, when the crystal was exposed to light, the density of photo-generated carriers would exceed the density of space-charge-injected carriers and that these photo-generated carriers would dominate the behavior of the crystal and lead to the ohmic behavior shown in Fig. 1.

DIRECT EVIDENCE OF SPACE-CHARGE

The interpretation in terms of space-charge-limited currents, with most of the space charge being trapped, suggested that one ought to be able to make a direct observation of this charge. Figure 2 shows the experimental arrangement used to make this test.

The crystal was held by spring action between two indium tipped electrodes. The crystal was mounted in a light-tight box and poised over a metal pan connected to an electrometer. Leads from the two electrodes were taken through the light-tight box to a source of voltage. One electrode was grounded. The other electrode could be connected to the positive or negative terminal of a dry cell or to ground. Finally, by an external mechanical arrangement the crystal could be released from the electrodes and dropped into the electrometer pan. The following observations were made:

(1) Both electrodes to the crystal were kept at ground and the crystal dropped into the electrometer pan. No charge was recorded. This indicated that the electrodes did not contribute any significant charge to the crystal by tribo-electric action.

(2) One electrode was grounded and the other electrode was held at *either* plus 100 volts or minus 100 volts relative to ground. A negative charge was recorded by the electrometer when the crystal was dropped into the pan. This test was subject to the criticism that charge from the electrodes might have in some way rubbed off onto the surface of the crystal. Hence the next test.

(3) One electrode was kept at ground. The other electrode was first tapped on the plus 100 volt terminal of the dry cell and then returned to ground, so that both electrodes were at ground just before the crystal was dropped to the electrometer pan. A negative charge was recorded. The same negative charge was recorded when the second electrode was tapped on the minus 100 volt terminal of a dry cell instead of the plus 100 volt terminal. The fact that the same negative charge was observed for either polarity of voltage applied to the crystal ruled out the possibility that the charge was due to a nonuniform resistance of the crystal. The internal charge would have changed sign in the latter instance.

(4) The magnitude of the charge was about half that expected from space-charge considerations and the geometry of the crystal. The uncertainty in area of contact of the electrodes could easily account for this discrepancy.

The procedure in item 3 was made possible by the fact that the space charge, forced into the crystal, was mostly trapped and remained in the crystal even when both electrodes were later grounded before the crystal was released.

DC CURRENT-VOLTAGE CURVES

Figure 3, curve I_0 shows an early measurement of the current through a crystal 5×10^{-3} cm thick when the crystal was kept in the dark. At each point the

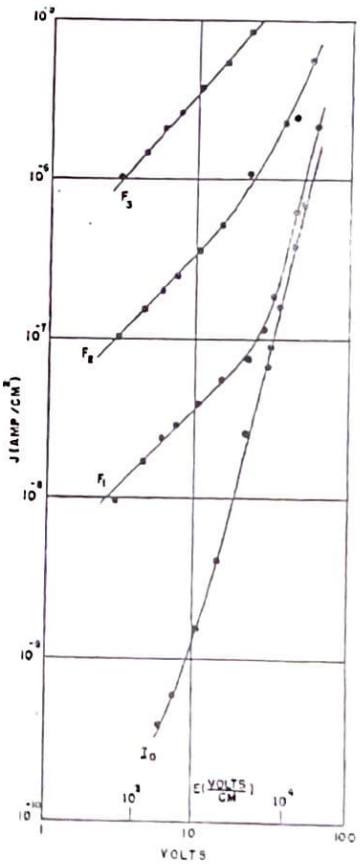

FIG. 3. Space-charge-limited current in an insulator. Dc measurement on a crystal similar to that of Fig. 2. I_0 is dark current curve and F_1, F_2, and F_3 are curves with increasing irradiation.

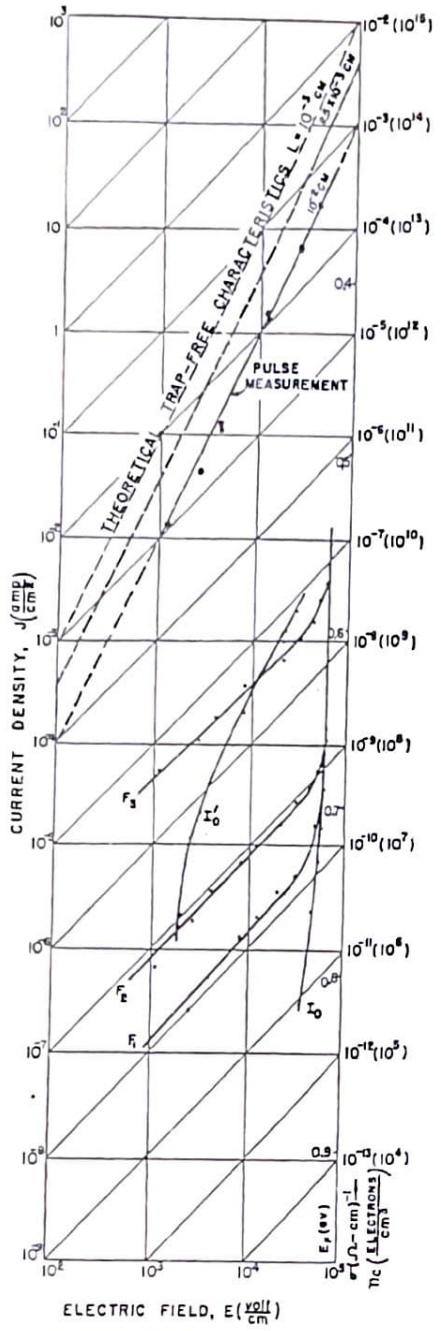

FIG. 4. Space-charge-limited current in an insulator. I_0' initial dark current curve after exposure to light, I_0 thermal equilibrium dark current curve. F_1, F_2, and F_3 curves obtained with different light levels on the crystal. The conductivity, density of carriers in conduction band n_c, and theoretical square-law curves calculated on basis of mobility $\mu = 100$ and dielectric constant $k = 10$ for CdS.

current was allowed to settle down to a reasonably stationary value. In the light of later experience it is not certain that sufficient time was allowed to get accurately stationary values. The curve, nevertheless, shows the high power dependence of current on voltage. In this case the current increases approximately as the fourth power of the voltage.

Also in Fig. 3 are shown curves marked F_1, F_2, and F_3. These curves were taken with small amounts of light on the crystal, the light intensity increasing from F_1 to F_3. They are significant in showing that for the same fields at which the dark current is increasing as the fourth power of the voltage, the photoconductive current increases only as the first power. These curves support the argument that the high power dependence of the dark current cannot be ascribed to collision ionization processes. It is also evident from Fig. 3, that at higher light intensities the transition from ohmic behavior to space-charge-limited current behavior occurs at higher voltages. This is consistent with the argument that the behavior is either ohmic or space-charge-limited depending on whether the volume generated carrier density or the injected carrier density predominates.

Figure 4 shows perhaps the most significant set of curves taken on a single crystal. The crystal thickness was 2.5×10^{-3} cm; the electrode area 5×10^{-4} cm^2. Again, the curve marked I_0 represents the current through the crystal in the dark. This time, however, sufficient time was allowed for the current to come to a reliably stationary value at each voltage. At the low current end, the time required was of the order of hours. The current increases as almost the 20th power of the voltage. Such a dependence[6] would be expected from traps uniformly distributed in energy at least in the range of 0.55 to 0.8 volt below the conduction band. This is the range of Fermi levels appropriate to the range of conductivities covered by curve I_0. It is also to be noted from this curve that the dark conductivity at fields below 10^4 volts/cm is well below 10^{-12} (ohm-cm)$^{-1}$.

The curves F_1, F_2, and F_3 were taken with the crystal exposed to increasing amounts of steady light. They show again, as in Fig. 3, the ohmic behavior obtained when the volume generated carriers exceed the space-charge injected carriers. They offer additional evidence that the electric fields (10^3 to almost 10^5 volts/cm) are not sufficient to cause collision ionization or even to significantly alter the "electron temperature."

TIME DEPENDENT CURRENTS

The curve I_0' of Fig. 4 has an important bearing not only on the interpretation of the present work but also on the interpretation of similar data reported by Böer.[7] Curve I_0' was taken with the crystal in darkness,

[6] A. Rose, following paper [Phys. Rev. 97, 1538 (1955)].
[7] K. W. Böer and U. Kümmel, Z. Naturforsch. 9a, 177 (1954).

but within a few minutes after it had been exposed to room light. Its particular shape is not to be emphasized since that depends on how rapidly the curve is taken. What is to be emphasized is that the I_0' curve lies well above the I_0 curve and that the I_0' curve represents a higher than ohmic dependence of current on voltage. A further significant fact is that if one waited long enough in taking this data, each point on the I_0' curve would have settled down to the I_0 curve.

The I_0' curve may be understood as follows. After the crystal has been exposed to room light and is put in darkness, thermal equilibrium is not immediately established. The conductivity decays roughly as t^{-1} and so decays more slowly as the conductivity decreases.[8,9] During this decay process many of the higher-lying trapping states are filled. A short-hand way of describing the electron distribution is to say that the steady-state Fermi level[10] is closer to the conduction band than its final equilibrium position.[9] Under these circumstances, the space-charge electrons are injected through the conduction band into levels closer to the conduction band so that a larger fraction of the electrons remain free. This accounts for the currents of the I_0' curve being higher than those of the I_0 curve. In the final steady state condition, the currents and electron distribution of the I_0' curve settle down into those of the I_0 curve. This settling process which would occur in the dark with no voltage applied to the crystal is considerably hastened by application of a voltage since the rate of approaching equilibrium is proportional to the free electron density. It is this hastening of the decay that accounts in large part for the ac observation described at the beginning of this paper, namely, the increase in resistance at a given voltage resulting from the previous application of a higher voltage. Curve I_0' was taken rapidly with increasing voltages. When the voltage range is retraced from high to low voltages a curve is obtained lying considerably below I_0' and close to I_0.

The last statement describes also the character of current-voltage curves reported by Böer for CdS crystals. Böer's interpretation is diametrically opposed to that taken here. He states that the high currents of the I_0' curve are a result of the emptying of traps by the applied field, either through collision ionization or field emission from traps; in brief, a field-induced "glow curve." What is common to both interpretations, Böer's and ours, is that the crystal after exposure to light, is left in a nonequilibrium condition and that, to approach equilibrium, electrons must recombine with deep-lying trapped holes created by previous optical excitation. In Böer's interpretation the electrons that recombine with the deep-lying holes come from higher-lying states and must first be excited into the conduction band by the applied field before recombining. While they are in the conduction band they would account for the excess currents of curve I_0'. As they drop into the deep-lying holes, the curve I_0 would be approached.

Our interpretation is that up to fields of about 5×10^4 volts/cm, the field is not strong enough to excite electrons from high-lying states into the conduction band. The electrons that recombine with the deep-lying holes are those that are injected into the conduction band from the cathode. The injection into the conduction band provides the initially high currents. The recombination of conduction electrons with deep-lying holes accounts for the approach to the low currents of curve I_0. The approach to I_0 is accelerated by the increased density of conduction electrons injected by the applied field. This approach takes place even in the absence of an applied field but much more slowly consistent with the low density of conduction electrons. The chief evidence for the present interpretation in terms of space-charge-limited currents is the fact that the electric fields are too low to extract electrons from or to ionize traps. Evidence of nearly equal importance, however, is contained in the curve marked "pulse measurement" in Fig. 4.

Pulse Measurements

Since space charge is injected first into the conduction band and *then* becomes largely trapped, one might expect to see the high theoretical currents characteristic of a trap-free solid if one looked fast enough after applying a voltage. Such a measurement was made using a voltage pulser[11] and an oscilloscope. Currents several orders of magnitude higher than those of curve I_0 were observed but they still fell far short of the theoretical curves shown in Fig. 4, both in magnitude and form. Since the oscilloscope could not resolve time better than about 100 microseconds, it was felt that some of the charge might be trapped before one could see its contribution to the pulsed current.

It was found, however, that if a small amount of steady bias light were used, much higher values of pulsed current could be observed. The curve marked "pulse measurement" in Fig. 4 was taken in this way. Two characteristics of this curve are immediately striking. The magnitudes of the currents are close to the theoretical values for a trap-free solid computed from Eq. (1), using a mobility of 100 cm²/volt sec. Especially at the lower fields these currents are well over 8 powers of ten higher than those of curve I_0. At the high end, the current densities reach 20 amperes/cm². The second significant characteristic is that the shape of this curve quite accurately follows the square-

[8] R. W. Smith, RCA Rev. 12, 350 (1951).
[9] A. Rose, RCA Rev. 12, 362 (1951).
[10] The steady-state Fermi level is defined by the relation $n=N_c\exp(-E_f/kT)$, where n is the density of free electrons, N_c is normally about 10^{19} at room temperature, and E_f is the energy difference between the bottom of the conduction band and the steady-state Fermi level.

[11] We are indebted to Dr. L. S. Nergaard for the use of this pulser which he had designed for use in oxide cathode work.

law dependence on voltage required by Eq. (1). Not only does the curve satisfy the magnitude and form of space-charge-limited currents in a trap free solid but it implies also two other important conclusions. The accurate square-law dependence on voltage means that in this range of fields, up to 5×10^4 volts/cm, the mobility is not field-dependent. The electron temperature does not depart significantly from the crystal temperature. Also, for currents up to 20 amperes/cm² the indium contacts are still ohmic, that is, they still supply a reservoir or excess of electrons at the metal-insulator interface. This must mean that the Fermi level of the indium metal is within about three-tenths of a volt of the conduction band of the CdS crystal.

Traces of the transient currents obtained with pulsed voltages are shown in Fig. 5. The decay time for these currents is of the order of a millisecond. This is also the decay time observed at high light intensities for photocurrents in this same crystal. The traps into which the space-charge electrons and the photoelectrons decay are either the same or at least have the same capture cross section. This cross section was computed from the relation $s=(\tau v n)^{-1}$ cm² to be 10^{-19} cm². τ is the decay time, 10^{-3} second; v the thermal velocity of an electron, 10^7 cm/sec; and n is an estimate of the *total* number of traps into which electrons may decay, 10^{16}/cm³.

TEMPERATURE DEPENDENCE

The injection of space-charge electrons into an insulator transforms it into a semiconductor of increasing conductivity as the voltage is increased. At any given voltage the current should vary with temperature in a similar fashion as for any semiconductor having a conductivity of the same order of magnitude. The total space-charge in the crystal should not vary with temperature; but the fraction of space-charge that is free should in general increase exponentially with increasing temperature. In reference 6 the following relation was obtained to describe the effect of temperature on the form of the current-voltage curve:

$$I \propto V^{(T_c/T)+1}.$$

T_c is a characteristic temperature describing the distribution of traps in energy. Small values of T_c are associated with trap distributions varying rapidly with energy, while large values of T_c approximate a slowly varying trap distribution. From Eq. (2), one would expect a steeper current voltage curve at lower temperatures. Figure 6 shows three current-voltage curves taken at different temperatures. Qualitatively the currents are smaller for lower temperatures, and the curves steeper at lower temperatures in accordance with the expected behavior. Quantitatively, the ratio of currents between the 371°K and the 300°K curves are of the right order. The currents of the 77°K curve should be much lower. It is possible that the electron distribution in the crystal did not come into thermal equilibrium with the crystal temperature. At these low carrier densities, approach to thermal equilibrium may easily require hours or days.

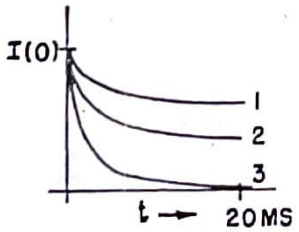

FIG. 5. Current in CdS crystal due to square voltage pulses of 20-millisecond duration. Curves 1, 2, and 3 correspond to initial currents I_0 of 6×10^{-6}, 2×10^{-4}, and 3×10^{-3} amp respectively.

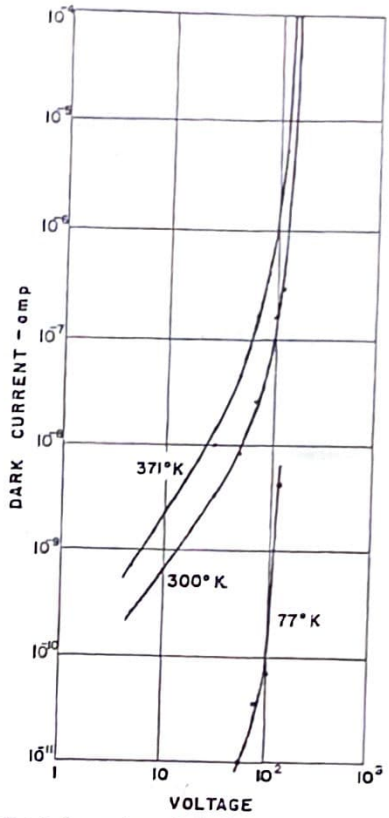

FIG. 6. Space-charge-limited current in CdS as a function of temperature.

COMPARISON OF SPACE-CHARGE-LIMITED CURRENT WITH PHOTOCONDUCTIVE CURRENT

The energy distribution of traps determines the form of the current-light curve in photoconductivity measurements and also the speed of response. The energy distribution of traps also determines the form and magnitude of the space-charge-limited current-

voltage curves. Conversely, from the observed curve forms and speeds of response one should be able to obtain the energy distribution of traps. A pertinent question is whether the trap distribution determined in the same crystal by the two independent methods of space-charge-limited currents and photoconductive currents shows any self consistency. Table I summarizes the appropriate relations derived in references 6 and 9.

In the limit for large values of T_c, the trap distribution becomes uniform in energy, the space-charge-limited current increases exponentially with voltage and the photocurrent increases linearly with light intensity. Figure 4 shows the exponential dependence of space-charge-limited current on voltage. Figure 7 shows the linear dependence of photo current on light intensity. Both sets of data were taken on the same crystal through the same electrodes. Both sets of data are consistent with a uniform or near uniform energy distribution of traps in the range of 0.5 to 0.8 ev below the conduction band.

The trap density computed from the space-charge-limited curve I_0 of Fig. 4 was 5×10^{12} traps/cm³ in an energy range of kT at 0.7 volt below the conduction band. From photoconductivity measurements at the light levels of curves F_1 and F_2 of Fig. 4, the trap density was computed to be 0.5×10^{12}/cm³ per kT at the same depth below the conduction band. Of the two estimates the space-charge-limited computation is likely to be the more reliable since the character or capture cross sections of the traps do not enter in. In the photoconductivity measurement, only those traps are measured from which thermal excitation occurs rapidly enough to keep pace with the decaying photocurrent (see reference 9).

TABLE I. The energy distribution and density of traps derived from data on photoconductivity and from data on space-charge-limited currents.*

	Space-charge-limited currents	Photoconductivity
Form of current curve	$I \propto V^{(T_c/T)+1}$	$I \propto F^{T_c/(T+T_c)}$
Trap density in range of kT near the Fermi level	$\frac{\Delta Q}{\epsilon}$	$\frac{\tau_\theta}{\tau}n_e$

* Notes: T_c defines the trap distribution in the relation $n_t \propto \exp(-E/kT_c)$, where n_t is the trap density at E volts below the conduction band. F is the number of optical excitations per second. ΔQ is the space charge forced into the crystal when the voltage is increased by an amount sufficient to double the current. τ_θ is the observed speed of response of the photoconductor. τ is the lifetime of a free carrier in the conduction band. n_e is the density of free carriers at which τ_θ is measured.

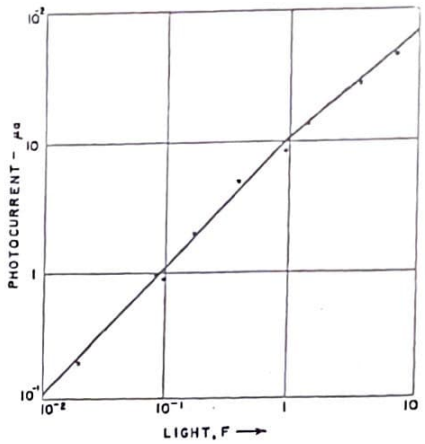

FIG. 7. Photocurrent-light (F) characteristic of CdS crystal.

The time of response at F_1 was 50 seconds and at F_2 was 6.6 seconds. At high light intensities the time of response leveled off at a value of 10^{-3} second, which should be close to the true lifetime of a carrier in the conduction band.

CONCLUDING REMARKS

Currents far in excess of ohmic currents have been measured through thin insulating crystals of CdS in the dark. A set of experiments has been described that identify these currents as space-charge limited currents in a solid. These experiments include the direct measurement of the space charge; the matching of the form and magnitude of the theoretically expected current in a trap-free solid by using pulsed voltages; the analysis of the much lower steady-state currents in terms of a trap distribution that is consistent with independent photoconductivity measurements; and the confirmation that these currents can be obtained at fields too low to cause collision ionization or field emission from traps. The fact that the steady-state space-charge-limited currents are more than eight powers of ten lower than those in a trap-free solid (Fig. 4) is evidence that space-charge-limited currents represent one of the most sensitive tools for measuring the presence of traps, especially in those insulating crystals that approach a high degree of perfection.

CAPITULO 4

Corrientes de inyección limitadas por carga espacial

EL ANÁLISIS DE corrientes de inyección en un aislante se hizo importante, en primer lugar, en la tecnología de tubos al vacío. El cátodo de emisión termoiónica se comporta como un contacto óhmico para el aislante, que en este caso es el vacío. El vacío es eléctricamente neutro en ausencia de electrones. Un flujo de corriente significa un flujo de electrones que siempre llevan cargas propias. Es por esto que hay una carga espacial cuando se hace pasar una corriente. El número de electrones que pueden ocupar el espacio entre el cátodo y el ánodo se determina por consideraciones electrostáticas. Esto impone una limitación en la corriente. El problema es discutido en libros de texto [1], pero un tratamiento más simple fue dado por Rose [2], cuyo análisis seguiremos aquí. En un diodo al vacío, el cátodo y el ánodo juntos forman un condensador de valor C. Bajo una tensión V hay una carga total Q:

$$Q = CV. \tag{4.1}$$

Como siempre, la tensión indica la presencia de cargas separadas negativas y positivas en números iguales. Las cargas positivas son el resultado de una falta de electrones y están en la superficie del ánodo. Las cargas negativas son electrones, los cuales pueden estar en el vacío en tránsito entre cátodo y ánodo. Un condensador de placas paralelas tiene:

$$C = 10^{-12}/4\pi L \text{ farad/cm}^2 \tag{4.2}$$

para una separación L, entre las placas. Es una buena aproximación suponer que esta capacidad también describe aquí la situación, en la cual los electrones están distribuidos entre las placas. Entonces:

$$Q = CV = 10^{-12}\, V/4\pi L \text{ coulomb/cm}^2. \tag{4.3}$$

La corriente I, depende del tiempo de tránsito T, de los electrones y de la carga total:

$$I = Q/T = 10^{-12}\ V/(4\pi L^2/\bar{v})\ \text{coulomb/cm}^2. \tag{4.4}$$

Aquí $\bar{v}$ es la velocidad media del electrón en tránsito entre las placas y sigue la relación:

$$\bar{v} = 3 \times 10^7\ V^{1/2}\ \text{cm/seg}. \tag{4.5}$$

Consecuentemente,

$$I = 2.4 \times 10^{-6}\ (V^{3/2}/L^2)\ \text{amp/cm}^2. \tag{4.6}$$

La ecuación 4.6 se denomina "la relación de Child-Langmuir" y se caracteriza por la potencia ⅜ que relaciona la corriente con la tensión.

El nuevo aspecto del problema de transporte en los sólidos es que en éstos, los portadores tienen una movilidad propia y no funcionan como partículas libres. El primer caso en el cual la movilidad se introdujo fue en el movimiento de iones en un gas, discutido por J. J. Thomson [3], en relación con las experiencias de Rutherford. Así, en los sólidos:

$$T = L^2/V\mu. \tag{4.7}$$

y nuevamente,

$$I = Q/T = (10^{-12}/4\pi)\ (\varepsilon V^2\mu/L^3)\ \text{amp/cm}^2. \tag{4.8}$$

Esta es la ley de Child-Langmuir para sólidos y fue encontrada por Mott y Gurney [4].

El electrón no se mueve como partícula libre. La movilidad hace constante su velocidad en un campo uniforme. La diferencia se manifiesta en el hecho de que la corriente en la ecuación 4.8 depende de la segunda potencia de V y no de ⅜. La corriente es bastante grande. Por ejemplo, si $\varepsilon = 4\pi$, $\mu = 100$ cm^2/V-seg, $L = 10^{-2}$ cm, $V = 100$ V, $I = 1$ amp/cm^2, entonces el aislante se transforma en un buen conductor. Así, hay dos cosas que protegen aislantes contra este caso. La primera es que los contactos óhmicos rara vez se hallan entre materiales en los cuales la banda prohibida es grande. Si no existen contactos óhmicos, no hay corriente de inyección. Esto fue reconocido primeramente por Rose [5]. Los argumentos empleados para llegar a la ecuación 4.8 suponen que los portadores inyectados se mueven libremente con movilidad μ, característica de portadores libres. Lo que acontece en muchos casos es que los portadores caen en trampas, centros localizados, en los cuales no pueden caminar. Los portadores de las trampas contribuyen a la carga espacial, pero no contribuyen a la conductividad. Es conveniente dividir las trampas en dos categorías:

trampas rasas * y *trampas profundas*. Los electrones en las trampas rasas están en equilibrio termodinámico con la banda de conducción. Asimismo, tales electrones están presos en las trampas, pero tienen oportunidad de salir por la banda de conducción. De esta manera, cada electrón pasa una cierta parte de su vida preso en una trampa y el resto moviéndose libremente en la banda de conducción. El equilibrio se mantiene por la acción de la agitación térmica y el intercambio de energía entre electrones y átomos del cristal. Llamaremos θ a la fracción de electrones libres en la banda de conducción. La corriente total para un valor V dado se hace θ veces la corriente de la ecuación 4.8 porque sólo los electrones libres contribuyen.

$$I = (10^{-12}/4\pi)\,(\theta\varepsilon V^2\mu/L^3)\ \text{amp/cm}^2. \tag{4.9}$$

Esta es la corriente limitada por la carga espacial en presencia de trampas rasas. En presencia de trampas profundas, existe una situación diferente. Las trampas profundas se definen como aquellas en las cuales los electrones están fuera de equilibrio con la banda de conducción. Generalmente, el nivel de energía del electrón en una trampa profunda queda bastante abajo del fondo de la banda de conducción. La agitación térmica no es suficiente para excitar el electrón a la banda de conducción dentro del tiempo del experimento. No hay un cambio de electrones entre las trampas y la banda de conducción. Decir que los electrones inyectados no dan origen a ninguna corriente, sino solamente a la carga espacial, constituye una buena aproximación. La corriente que existe se debe al pequeño número de electrones libres en el cristal neutro. Por esta razón, la corriente sigue la ley de Ohm hasta que haya suficientes electrones inyectados para llenar todas las trampas. Cualquier aumento adicional de la tensión aplicada inyecta más electrones, pero no hay más lugares en las trampas. Por consiguiente, todos los electrones "nuevos" están libres, listos para contribuir a la corriente. La tensión que se produce cuando todas las trampas están llenas, se llama *tensión de límite de las trampas llenas*. Más allá de este límite, un aumento de la tensión, por pequeño que sea, causa un aumento repentino en la corriente. Como las trampas ya no representan ningún papel, el valor de la corriente —más allá del límite de las trampas llenas— sigue la ecuación 4.8, que vale en ausencia de trampas. Un ejemplo de este fenómeno se muestra en la figura 4.1 de R. W. Smith [6]. En este caso, el material es CdS, y la figura indica el logaritmo de la densidad de corriente contra el logaritmo del campo aplicado. La corriente en lo oscuro para campos bajos es demasiado pequeña para hacer mediciones.

* Trampa rasa significa que los electrones en estas trampas están en equilibrio termodinámico con la banda de conducción; esto es, no se hallan muy apartados del fondo de la banda de conducción. [N. del T.]

Por esta razón la muestra es iluminada con varias intensidades de luz, para introducir portadores libres. Hasta el límite de las trampas llenas a la derecha, sólo los portadores proporcionados por la luz contribuyen a la corriente y no los portadores inyectados. En el límite de las trampas llenas, la corriente aumenta repentinamente hasta el valor de la ecuación 4.8.

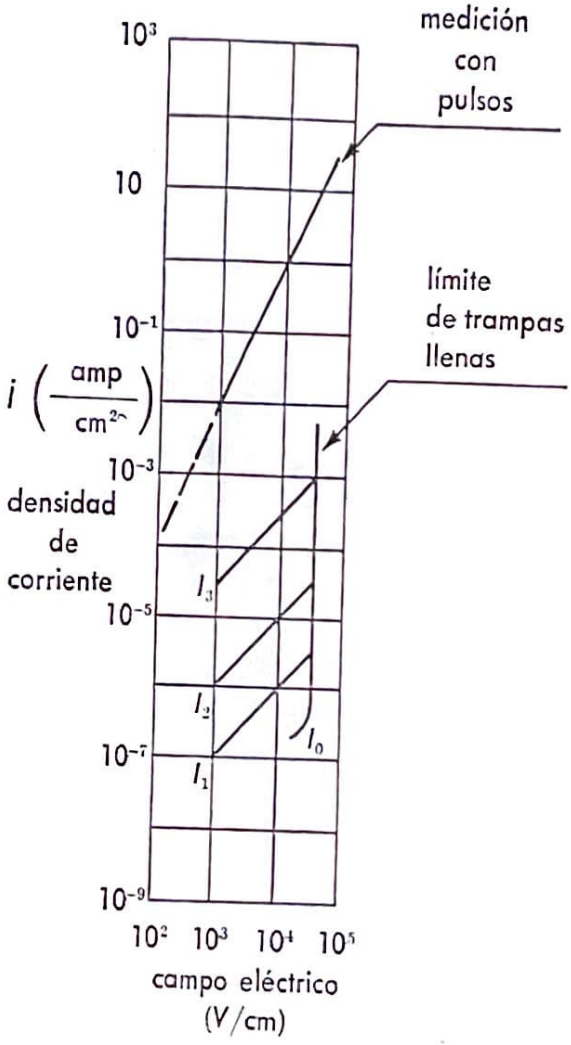

FIG. 4.1

Mediciones con impulsos cortos de tensión dan valores de la corriente, apropiados para una muestra sin ninguna trampa. Esto acontece debido a que un electrón libre precisa de un cierto intervalo de tiempo para encontrar una trampa. Mediciones hechas con tiempos más cortos están relacionadas con electrones libres. La figura 4.1 resume las características importantes de las corrientes de inyección en presencia de trampas profundas.

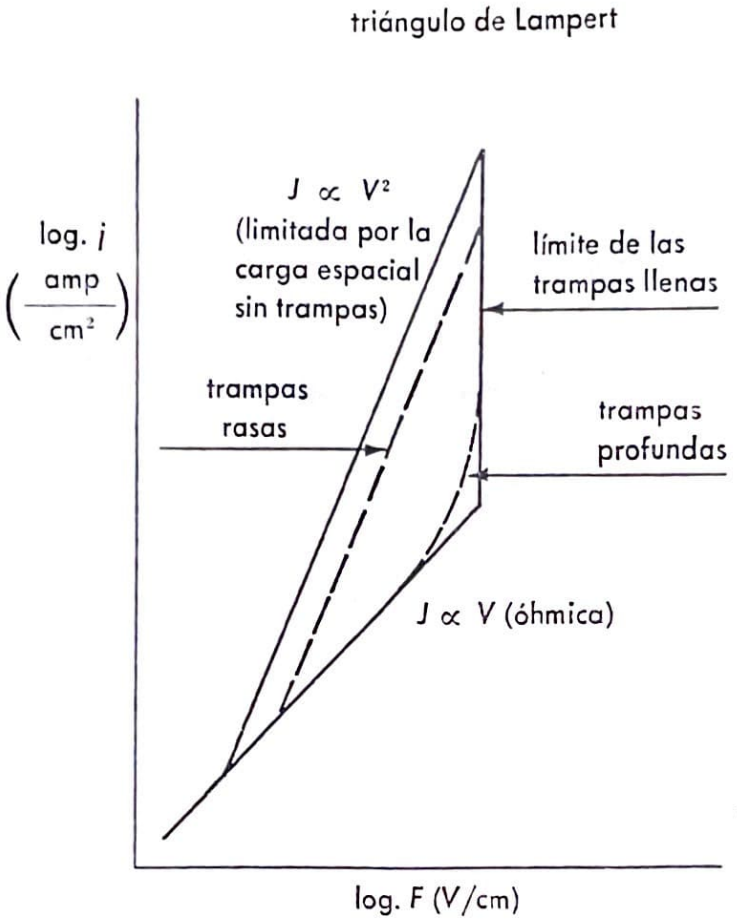

FIG. 4.2

Por otra parte, trampas rasas y profundas corresponden a dos límites de comportamiento. En la naturaleza existe una transición continua entre ambos límites. El trabajo de M. A. Lampert ofrece un esquema conveniente que resume todas las posibilidades al respecto de las corrientes de inyección. (La figura 4.2 muestra el triángulo de Lampert.) El logaritmo de la corriente es mostrado como función del logaritmo de la tensión, de acuerdo con la teoría. El modelo incluye todas las trampas que tienen el mismo nivel de energía, y se obtienen curvas que corresponden a varios valores de energía. La recta con pendiente 2 corresponde a la ecuación 4.8. La corriente nunca puede ser mayor para una tensión específica, que el valor dado por esta ecuación, porque esto representa el caso en el cual todos los portadores inyectados están libres. La recta de pendiente 1 representa el caso en el cual todos los portadores inyectados están presos en trampas profundas y, por tanto, no contribuyen en nada a la corriente. Ésta resulta sólo del movimiento de los portadores presentes en equilibrio en el cristal neutro, y sigue la ley de Ohm. La corriente nunca puede ser menor que la

corriente representada por esta recta, porque los portadores inyectados sólo pueden aumentar la corriente, pero no disminuirla. El límite de las trampas llenas aparece a la derecha. En presencia de cualquier combinación de trampas, la corriente real debe corresponder a un punto dentro del triángulo. Las rectas representan dos casos particulares, que son analíticamente tratables. La recta más alta representa trampas rasas (energía $E_a = 0.4\ eV$ abajo del fondo de la banda de conducción, $T = 25°$ C) y la otra representa trampas profundas ($E_a = 0.7\ eV$).

La corriente, en el caso de trampas rasas, corresponde a la corriente en ausencia de trampas, reducidas por el factor θ. En el caso de trampas profundas con E_a mayor que $0.7\ eV$, es evidente que los portadores inyectados no harán ninguna contribución a la corriente sino hasta el límite de las trampas llenas. Entre los aislantes, este caso es común. Hay muchos aislantes que perderían la capacidad de aislar si no tuviesen trampas en determinadas concentraciones. Los resultados antes mencionados se refieren, en gran parte, a los sistemas en estado de equilibrio o en estado de cuasiequilibrio, en el cual la ocupación de las trampas alcanza su estado final. Efectos transitorios en el camino del estado final también pueden dar información importante sobre el transporte.

EFECTOS TRANSITORIOS

Los efectos transitorios sirven para investigar el transporte en los aislantes, evitando los efectos de las trampas. Alternativamente, es posible emplear métodos similares, a fin de estudiar las propiedades de trampas bajo condición controlada. Los trabajos de Many, Weiss y Simhony [8] y el de Mark y Helfrich [9] sirven para ilustrar las ideas generales. Many, Weiss y Simhony usaron cristales de yodo. El contacto óhmico fue hecho mediante la iluminación de una superficie con luz fuertemente absorbida. Esto crea un reservatorio de electrones y hoyos libres en la superficie. En este caso, los portadores móviles son los hoyos. Un impulso de tensión de polaridad apropiado jala hoyos a través del cristal. Durante la primera onda de hoyos atravesando el cristal, los hoyos que toman la delantera sienten un campo diferente de aquel que se siente más tarde cuando todo el cristal está lleno de hoyos. Por tanto, la corriente depende del tiempo (véase figura 4.3). Dicha corriente pasa por un pico y después se aproxima a un valor constante. El resultado de esto, es bien explicado por la teoría que considera la carga instantánea y el campo correspondiente. La movilidad de hoyos tiene el valor de 0.7 cm^2/V-seg. En ciertas muestras, son evidentes los efectos debidos a trampas. Hay indicaciones de que la corrien-

te después del pico no se aproxima a un valor constante, sino que disminuye continuamente. Esto significa que los portadores están siendo presos por las trampas. Su contribución a la carga espacial permanece, pero su contribución a la corriente desaparece. Un análisis de los resultados indica, en muestras diferentes, concentraciones de trampas de 10^{11} cm^{-3} hasta 10^{14} cm; los niveles de energía permanecen entre 0.45 y 0.6 *eV*, respecto a la banda de valencia. La sección transversal de captura vale 10^{-20} cm^2.

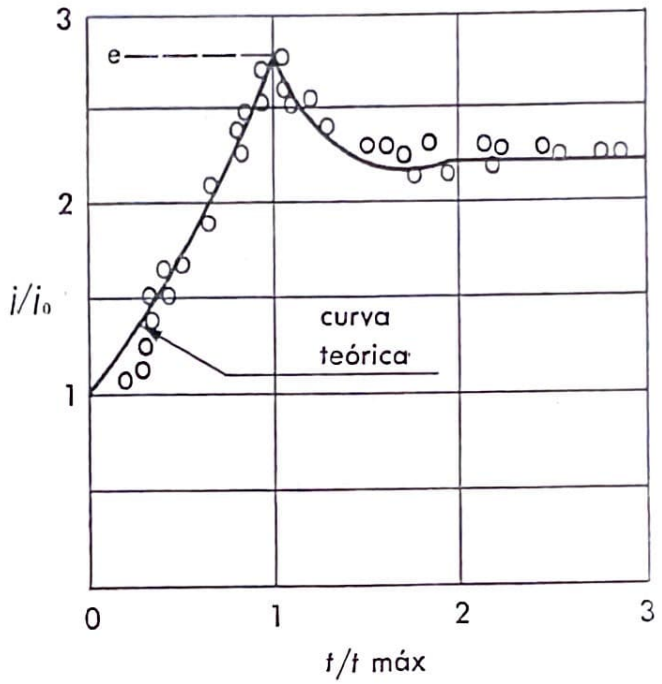

FIG. 4.3

Mark y Helfrich investigaron materiales orgánicos como antraceno, *p*-trifenol y *p*-cuatrofenol. El contacto óhmico fue hecho con una solución de yodo 1M NaI en agua. Esta combinación ofrece un reservatorio de hoyos. Hay corrientes de inyección limitadas por la carga espacial y una saturación del contacto cuando la corriente alcanza el valor aproximado de 10^{-7} amp/cm^2. Los efectos transitorios son similares a los encontrados para el yodo, incluyendo efectos de trampas. Para el *p*-trifenol, la movilidad es del orden de 2.5×10^{-2} cm^2/Vseg; y la concentración de trampas es del orden de 10^{12} cm^{-3}. La sección transversal de captura es igual a 4×10^{-17} cm^2.

Contactos de electrólitos sirven también para hacer contactos óhmicos en otros materiales. Por ejemplo, una solución de SO_3 en H_2SO_4 sirve para inyectar hoyos en azufre [10]. Una vez más, el comportamiento está dominado por las trampas que se hallan presentes en concentraciones del orden de 10^{15} cm^{-3}. Es conveniente hacer notar aquí la naturaleza

cáustica de las sustancias empleadas para hacer contactos óhmicos con los aislantes que tienen bandas prohibidas grandes. Esto refleja la conexión entre la estructura de las bandas de energía y los niveles de energía de electrones en los iones en solución.

DISTRIBUCIÓN DE CARGA Y CAMPO EN PRESENCIA DE TRAMPAS PROFUNDAS

El problema de distribución de carga y campo asociado con las corrientes de inyección es difícil de resolver en el caso general, en presencia de trampas de tipos diferentes. En ausencia de trampas, los electrones inyectados tienden a concentrarse cerca del cátodo donde el campo es mínimo. En presencia de trampas profundas, todos los electrones permanecen en las trampas para tensiones abajo del límite de las trampas llenas. La distribución de cargas es uniforme entre cátodo y ánodo. El campo es determinado por consideraciones electrostáticas. La ecuación de Poisson da la relación entre el campo F, y la carga espacial q, que aquí es igual a la concentración de las trampas llenas N_t, multiplicada por la carga del electrón:

$$\frac{dF}{dx} = \frac{d^2V}{dx^2} = \frac{4\pi N_t e}{\varepsilon}. \tag{4.10}$$

Refirámonos ahora al caso de inyección de electrones, en el cual el contacto inyector es el cátodo. Uno de los fundamentos de la teoría de la corriente de inyección radica en que el campo debe ser cero en el contacto inyector. Por tanto, supondremos aquí la misma cosa. Consideremos, pues, una lámina de aislante entre electrodos de placas paralelas separadas a una distancia d. En este caso, el problema tiene una sola dimensión. Medimos la distancia a lo largo del eje x y colocamos el cátodo en el punto $x = 0$, y el ánodo en el punto $x = d$. En la ecuación 4.10 todas las cantidades al lado derecho son independientes de x. Por consiguiente:

$$\frac{dV}{dx} = (4\pi N_t e/\varepsilon)x + C_1. \tag{4.11}$$

Por hipótesis,

$$F = -dV/dx = 0, \text{ en } x = 0 \text{ y } C_1 = 0.$$

$$V = (4\pi N_t e/\varepsilon)\,(\tfrac{1}{2}x^2) + C_2. \tag{4.12}$$

Consideramos el cero de potencial como el potencial en el cátodo o el punto donde $x = 0$, de manera que $C_2 = 0$. Entonces,

$$V(x) = (2\pi N_t e/\varepsilon)\, x^2, \tag{4.13}$$

y la tensión total aplicada es:

$$V(d) - V(0) = (2\pi N_t e/\varepsilon)\, d^2. \tag{4.14}$$

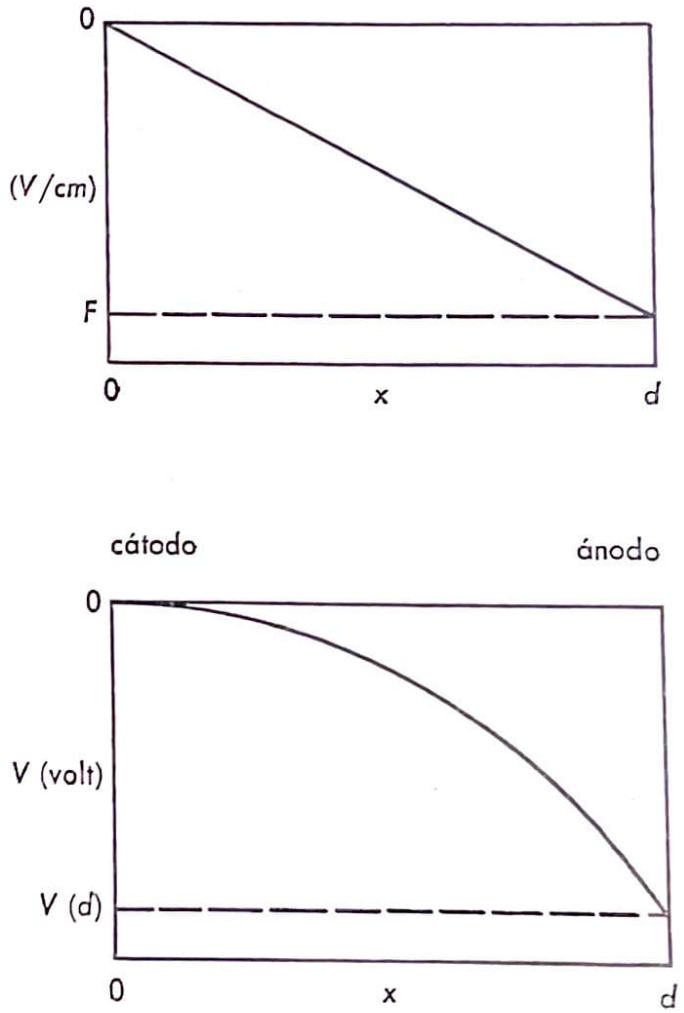

FIG. 4.4

La figura 4.4 muestra el potencial y el campo entre los electrodos como funciones de x. El campo es función lineal de x y el potencial es una función cuadrática. La misma distribución de potencial y campo también vale para la barrera de Schottky. El campo medio es igual a la mitad del campo máximo. En unidades prácticas:

$$V(x) - V(0) = 9.1 \times 10^{-7}\, N_t x^2/\varepsilon, \tag{4.15}$$

donde: N_t es el número de trampas llenas por cm^3, x está dado en centímetros, y ε es la constante dieléctrica normal. Los mismos argumentos son válidos para el caso en que los portadores son hoyos. Naturalmente, el signo del campo es opuesto.

Lecturas complementarias

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